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In Equations we Trust? Formula Learning Effects on the Exponential Growth Bias

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Abstract: This paper evaluates the possible benefits and drawbacks of the formal formula learning of compound growth as it pertains to eliminating, or at least reducing, the exponential growth bias in various household savings and debt decisions. In our main experimental study, we determine if the ability to calculate the simple compound savings formula only assists in its direct area of application with an available calculator, or if this knowledge extends into similar exponentially-based savings and debt decisions when either a calculator is prohibited or when the formula is unknown. In the process of tackling this research question, we develop a measure for the exponential growth bias that naturally extends over different tasks and parameter settings. Our findings suggest that learning the compound savings formula does much more than eliminate the exponential growth bias for individuals in the savings domain with an available calculator. In fact, we find evidence that these individuals post less biased savings and debt estimates in the absence of a calculator, suggesting that the knowledge of this formula may aid in developing a more general, intuitive grasp of exponential effects. On the other hand, we find that too much dependence on these formulas can have adverse effects, as a number of participants who knew the compound savings formula mistakenly applied a variation of it in the debt domain leading to insensible answers well above the initial loan balance.

Keywords: Behavioral Finance, Exponential Growth Bias, Amortization Bias, Financial Decision Making

JEL Classification: D14

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1. Introduction

Evidence of the exponential growth bias (EGB), defined by Stango and Zinman (2009) as “the tendency to linearize exponential functions when calculating them intuitively”, has consistently been demonstrated in various domains (Wagenaar and Sagaria, 1975; Wagenaar and Timmers, 1979; Keren, 1983; Benzion, Granot, and Yagil, 1992).¹ Lately, EGB has been extensively analyzed in the context of household finance, including the underestimation of compound savings growth and annual interest rates on credit cards (Stango and Zinman, 2009; McKenzie and Liersch, 2011; Almenberg and Gerdes, 2012; Song, 2012). EGB has also been evaluated within an experimental setting in the savings domain, where Eisenstein and Hoch (2007) find that 90% of respondents underestimate compound growth in savings questions. Levy and Tasoff (2015) also find “substantial” EGB in their incentivized experiment, noting that “subjects are largely unaware of their bias and undervalue assistance.” The potential consequences of EGB within the context of household finance are stated by Stango and Zinman (2009), who find that households with higher EGB tend to “borrow more, save less, and favor shorter maturities” compared to the less biased households.

In addition to making critical long-term savings decisions, many consumers have been placed in a position of making important debt decisions. This makes the basic understanding of loans and how they are amortized paramount. A lack of this basic understanding can potentially lead to unsustainable debt and higher costs of borrowing. According to Lusardi (2011), one in five Americans used a high-cost borrowing method (payday loan, pawn shops, etc.) between 2005 and 2009.² Stango and Zinman (2009) find that consumers demonstrate a payment/interest bias in which people show a systematic tendency to underestimate the interest rate of a particular

¹ Because of this “tendency towards linearity”, EGB is sometimes also referred to as “linear bias” in academic literature.

² It has also been shown that those possessing greater “debt literacy” - those who understand the basic concepts of debt - are more likely to avoid using these high-cost methods of borrowing and pay their credit cards in full (Lusardi and Tufano, 2009).

loan given the principal amount, monthly payment and maturity. Soll, Keeney, and Larrick (2013) find that people underestimate the time it takes to eliminate a debt based on a known monthly payment. To further extend the body of evidence on the neglect of exponential effects in the debt domain, we use a different debt-related task in our study: we ask participants to estimate the remaining balance on a loan at various points of a debt payoff schedule and observe a systematic bias towards linearity, which we call the “amortization bias”.

Given the immense ramifications of these savings and debt decisions, it is important to understand how teaching people about the effects of compounding interest decreases the bias, which has been previously analyzed (MacKinnon and Wearing, 1991; Eisenstein and Hoch, 2007; McKenzie and Liersch, 2011; Song, 2012; Soll, Keeney, and Larrick, 2013; Goda, Manchester, and Sojourner, 2014; Levy and Tasoff, 2015). For us, the natural next step in this process is to determine if learning the compound savings formula provides benefits that extend beyond its direct application in a simple savings scenario with an available calculator. Therefore, in our main study (Experiment 1), we explore two additional potential benefits of those who can correctly apply this basic formula. First, does the ability to calculate the formula improve judgment in simple savings situations where no calculator is available? Secondly, does this ability improve estimates outside of the saving domain, when a slightly more complicated exponentially-based debt question is provided and the formula is unknown? If this formula learning is shown to be effective, the findings could speak in favor of introducing a more formal learning curriculum, which is in contrast to the current popular prescriptions to improve financial decision making: financial literacy and personal finance education. These initiatives have recently been shown to be costly, investing billions of dollars annually, and largely ineffective in providing any long-term assistance in improving financial decisions (Fernandes, Lynch, and Netemeyer, 2014). If teaching the simple compound interest formula improves decision making in multiple scenarios, this method of learning could provide an effective and

inexpensive alternative to improving individual decision making. On the other hand, if this learning doesn't improve estimates outside of its direct application, we would have sufficient evidence on the limitations of this formal learning and can concentrate further research on exploring other alternative learning methods.

This paper also includes a follow-up experimental study (Experiment 2), where we explore the retention of this formula learning over time. We believe that if the formula learning proves to be effective in making better decisions in multiple scenarios, the benefits could remain somewhat limited if it cannot be adequately retained over time. Therefore, this follow-up study tests the “stickiness” of this formula learning by testing a specialized group of participants approximately 18-20 months after receiving extensive learning on discounting and the compound savings formula in a mandatory university course.

2. Main Hypotheses – Experiment 1

In our main study (Experiment 1), we test incoming freshmen at a top German university with various exponentially-based savings and debt questions. Our experimental setting allows us to distinguish between individuals who are capable of correctly calculating compound interest and those who are not. Instead of grouping individuals on their self-stated ability to correctly calculate compound interest, we assign groups based on the actual correctness of answers in a simple savings scenario with an available calculator. Accordingly, we refer to these two groups as “capable with a calculator” and “incapable with a calculator”.

Our first research question is whether those participants who are “capable with a calculator” could effectively overcome the “tendency to linearize” these compound savings estimates when calculators are prohibited and are thus forced to think about the answers intuitively. We don't believe that the bias will be completely eliminated in such a scenario and predict that a significant EGB will be observed for this group. Thus, our first hypothesis is:

H1 (EGB in the savings domain without calculators): For the “capable with a calculator” participants, an EGB will be observed in the savings domain when calculators are prohibited.

On the other hand, we believe that those in the “capable with a calculator” group will demonstrate a better intuitive understanding of exponential growth and provide less biased estimates than the “incapable with a calculator” group when tested with savings questions in the absence of a calculator. Presumably, someone who knows the equation on how savings exponentially compounds over time should be able to adjust their intuitive/linear estimates better than those who are unable to make the calculations. Hence, we hypothesize:

H2 (Less EGB in the savings domain without calculators): Compared to the “incapable with a calculator” group, the “capable with a calculator” group will provide significantly less biased savings estimates when calculators are prohibited.

In addition to testing the savings domain without a calculator, we also extend this analysis into the debt domain, where we ask a slightly more complicated exponentially-based debt question: What is the remaining balance on a given x -year loan after making payments for y -years?

In the debt domain, the formal derivation of the remaining balance B after n ($\leq N$) payments amounts to:

$$B = A \cdot \left[1 - \frac{(1+i)^n - 1}{(1+i)^N - 1} \right] \quad (1)$$

if the overall loan is to be paid back in N equal installments on A , the initial balance of a loan with an interest rate i . The equation (1) shows that the calculation of the remaining balance is driven by simple exponential effects: the principal reduction is just a quotient of two compound interest terms.³ However, the connection is not as obvious as in a simple savings scenario, where

³ The full derivation of equation (1) can be found in Appendix 1.

a present value PV is multiplied by an exponential factor $(1 + i)^t$ to obtain the final value FV of investing t periods at an interest rate of i :

$$FV = PV \cdot (1 + i)^t \quad (2)$$

From a technical perspective, the calculation in the amortization scenario (1) is not much more complicated than the calculation in the simple compound savings scenario (2). However, we are confident that the actual formula, or an alternative simple heuristic such as the Rule of 72, which estimates an investments doubling time, will not be widely known by the participants in this domain.⁴ This essentially puts each individual on a common ground where we can evaluate if an improved intuition and understanding can be extended from the compound savings formula learning to other domains.

In this domain, we believe that we will observe first-findings of what we call the amortization bias, which is the tendency to linearize the remaining balance on a loan at various points in time. Furthermore, we expect that the general understanding of exponential effects of the “capable with a calculator” group would lead to higher, and less biased, estimates in the debt domain, both with and without an available calculator. Thus, the fourth and fifth hypotheses are derived as:

H4 (Amortization bias in the debt domain): In the debt domain, the overall participant pool will be systematically biased when estimating the remaining balance on a loan at various points of their debt payoff schedule.

H5 (Less amortization bias in the debt domain with and without calculators): The “capable with a calculator” group of participants that can calculate the correct savings answer will provide less biased estimates in the debt domain than the “incapable with a

⁴ The rule of 72 estimates how many times an investment will double, formally given by the time in years multiplied by the annual interest rate divided by 72. This heuristic is largely unknown in Germany, even within the academic community.

calculator” group, both (5a.) when calculators are prohibited and (5b.) when calculators are available.

3. Measuring Bias Size

One of the major challenges for this type of research is that we have to compare and aggregate bias size across tasks and for different parameters. Therefore, we seek a measure for the exponential growth bias that naturally extends over different tasks and can be calibrated in a meaningful way. Wagenaar and Timmers (1979) and Stango and Zinman (2009) have suggested measuring the strength of the exponential growth bias in the savings domain by inserting a parameter θ into the relation between present and final value to make it:

$$FV = PV \cdot (1 + i)^{(1-\theta)t} \quad (3)$$

An unbiased answer is given for $\theta = 0$. Participants with a θ greater than zero show a typical exponential growth bias and provide a future value FV that underestimates the effects of compound interest for a given PV , t , and i .

This measure is able to distinguish between individuals who are unbiased and those who show an exponential growth bias (or a reverse bias). It can also rank participants’ answers by bias size for a given scenario of i and t . We also like the general approach of attaching the bias measure to the accumulation factor f in the equation

$$FV = PV \cdot f_{i,t}(\theta) \quad (4)$$

and to measure the bias as $\theta = f_{i,t}^{-1}\left(\frac{FV}{PV}\right)$. Such an approach seems promising for an extension of the concept to other domains and tasks in which exponential components are the key drivers of the calculations (as we have seen in equation (1) for our amortization problem). If bias size is attached to the distortion of these exponential components, we have a simple and canonical

way of relating bias size to each other across scenarios and tasks even though the absolute response scales can be very different.

Unfortunately, the very specific factor $f_{i,t}(\theta) = (1 + i)^{(1-\theta)t}$, used in the previous literature, turns out to be problematic if we want to compare the bias size for different parameter combinations within task and even more so across different tasks.

The problem arises because for this specific $f_{i,t}(\theta)$ the typical underestimation of the final value of a long-term investment is not modeled as a (partial) neglect of the higher order compound interest components of the total interest but as a distortion of the perceived investment time. The naïve investor who completely neglects compound interest and believes that a \$100 endowment will grow to \$120 over five years at a 4% interest rate is assigned a θ of 0.07, because he behaves like an investor who fully appreciates compound interest but collapsed the relevant time to a period of $((1 - .07) \cdot 5) = 4.65$ years. The same naïve investor would get assigned a θ of 0.42 when confronted with a 20 year investment at 10% interest rate, because his naïve estimate of the final value, \$300, is obtained for an investment of $(0.58 \cdot 20) = 11.6$ years while perfectly appreciating all compound interest components.

A measure that assigns such different θ values to an individual who follows a consistent and canonical strategy (of completely ignoring compound interest components), doesn't seem to be very suitable for our purpose and for any research that considers the exponential growth bias to be a personal trait rather than a scenario dependent bias.⁵

⁵ Levy and Tasoff (2015) point towards an even more obvious problem of the approach: an exponential growth biased individual ($\theta > 0$) would be predicted to also misjudge the return in a one-period setting, i.e. the one-year return of an investment with (annual) interest rate i would be estimated to be smaller than i .

Therefore, we extend the set of basic properties that have been claimed by Stango and Zinman (2009) and are fulfilled by their approach $f_{i,t}(\theta) = (1 + i)^{(1-\theta)t}$ and propose that a convincing measure should furthermore have the following properties:⁶

- (a) It should be calibrated not only for the perfect exponential decision maker (i.e. have the property $f_{i,t}^{-1}((1 + i)^t) = 0$ but also for a completely naïve decision maker who fully ignores the compound interest components. An intuitive calibration would be to claim: $f_{i,t}^{-1}(1 + t \cdot i) = 1$.
- (b) It should be able to assign a θ -value to any meaningful estimate, i.e. $f_{i,t}^{-1}$ has to be defined on the complete set of meaningful answers, and it should be monotonic.
- (c) It should have properties (a) and (b) not only for the standard savings scenario from equation (4), but also for other domains and tasks, in particular the debt amortization scenario we consider in this research.

In Appendix 2, we discuss various potential measures with respect to these properties and find a simple geometric mixture of the linear and the exponential return, i.e. an accumulation function $\check{f}_{i,t}(\theta) = (t \cdot i)^{(\theta)} \cdot ((1 + i)^t - 1)^{(1-\theta)} + 1$ to be appropriate.

It has the desired properties:

$$\check{f}_{i,t}^{-1}((1 + i)^t) = 0 \quad (5)$$

$$\text{and} \quad \check{f}_{i,t}^{-1}(1 + t \cdot i) = 1 \quad (6)$$

and is able to assign a bias size θ to any answer $FV > PV$ in the savings domain.⁷

⁶ A different set of properties for a suitable growth bias measure is suggested by Levy and Tasoff (2015). They do not search for a measure that extends over different scenarios and domains, however, but analyze a framework, in which interest rates can vary over time.

⁷ We would not consider it a sensible answer if an investor estimated that a \$10,000 endowment PV, invested at an interest rate of $i=5\%$ for 3 years, would “grow” to a final value FV of \$9,900.

It also behaves nicely in the amortization scenario. If we follow the general approach of attaching the bias measurement to the exponential components and generalize the amortization equation (1) to become

$$B = A \cdot \left[1 - \frac{\check{f}_{i,n}(\theta) - 1}{\check{f}_{i,N}(\theta) - 1} \right] \quad (7)$$

we can write:

$$B = A \cdot \check{g}_{i,n,N}(\theta) \quad (8)$$

with

$$\check{g}_{i,n,N}(\theta) = 1 - \frac{\check{f}_{i,n}(\theta) - 1}{\check{f}_{i,N}(\theta) - 1} \quad (9)$$

The derived function $\check{g}_{i,n,N}(\theta)$ that is used to determine the bias size in the amortization scenario as: $\theta = \check{g}_{i,n,N}^{-1}\left(\frac{B}{A}\right)$ has the same nice calibration properties as $\check{f}_{i,t}(\theta)$. It assigns a bias of 0 to a perfect exponential estimate, i.e.

$$\check{g}_{i,n,N}^{-1}\left(1 - \frac{(1+i)^n - 1}{(1+i)^N - 1}\right) = 0 \quad (10)$$

and a bias size of 1 to a completely naïve debtor who assumes the remaining balance to decrease linearly in time:

$$\check{g}_{i,n,N}^{-1}\left(1 - \frac{n}{N}\right) = 1 \quad (11)$$

The function $\check{g}_{i,n,N}(\theta)$ is furthermore monotonic and can assign a bias size θ to any answer $B < A$ in the debt domain.⁸

⁸ Again, we would not consider it a sensible answer if an investor estimated that a debt amount of \$200,000 in a 20-year amortization scheme has “decreased” to an outstanding balance of \$210,000 after 5 years.

4. Experimental Setup – Experiment 1

4.1 Participants

In this experiment, we tested 273 first semester undergraduate students, 121 males and 152 females, who were in their first few weeks of lectures in the fall semester at the University of Muenster. The median participant age was 19 years, ranging from 17 to 31. This experiment was conducted in an experimental lab and was given in German.⁹

4.2 Experimental Design

In round 1, we initially ask the participants to estimate 21 questions (15 savings and 6 debt). In this round, we prohibit the use of calculators, but allow the use of pens and paper provided in the experimental lab. We use a question structure in the savings domain that is consistent with Eisenstein and Hoch (2007). These questions include:¹⁰

- Nine different prospective savings questions – which asks how much an initial investment of \$10,000 grows over x -years earning a constant $y\%$ annual interest rate.¹¹
- Four different retrospective savings questions – which asks what one-time investment is needed to reach a savings goal of \$100,000 after x -years while earning a constant annual interest rate of $y\%$.

In these two cases, the correct application of the exponential formula can be determined by

$FV = PV * (1 + i)^t$ (prospective scenario) and $PV = FV / (1 + i)^t$ (retrospective scenario).

- Six different long-term debt questions – These questions ask:

⁹ While Business and Economics majors take first semester courses in English, not all first semester students take mandatory English courses.

¹⁰ See Supplement A for the full wording of each type of question.

¹¹ Additionally, we asked two of the same prospective questions at the conclusion of the round as we wanted to investigate the consistency of within-subject answers. However, for purposes of clarity in regards to our main research question, we do not publish the results in this paper.

Today, you borrow \$_____ for ____ years, paying a yearly fixed interest rate of _____%, agreeing to pay off the entire loan plus interest by making _____ equal monthly payments.

Assume all payments have been made on time and no additional payments have been made.

After making payments on this loan for _____ years (____ payments), what is the remaining balance of the initial loan? Please provide your best estimate.

The three-by-three (two-by-three) question vector in the prospective savings (debt) domain, yearly interest rate by time in years (remaining on loan), allows for data on three points of the exponential growth curve.¹² Upon completion of the first round of questions, the participant received a non-programmable, scientific (Olympia LCD-8110) calculator from the experimenter and retook six savings questions (four prospective and two retrospective) as well as four long-term debt questions.¹³ Participants were prohibited to use the internet or any other personal devices while completing the experiment. An overview of the overall experiment is shown in Table 1, and the full parameterization of this experiment is displayed in Appendix 3. We programmed this experiment to prohibit and post an error message for most answers that were given outside of the sensible range, i.e. less than the initial savings amount or greater than the initial loan amount of \$100,000.¹⁴ This error message persisted until the participant posted an answer within the sensible range. This design allows for a potential opportunity to analyze how individuals who post insensible answers adjust their estimates into a sensible range.

¹² An abbreviated two-by-two vector is utilized for the four retrospective questions.

¹³ Each set of questions were given in a random order.

¹⁴ Answers less than \$0 did not post an error message, but are excluded from the analysis.

Table 1. Experiment overview - Experiment 1. This table outlines the overall activity in Experiment 1, by round.

Round 1 (without calculator)	
9 Savings Questions (Prospective) 4 Savings Questions (Retrospective) 6 Debt Questions 2 Redundant Savings Questions (Prospective)	<i>Random Order*</i>
Round 2 (with calculator)	
6 Savings Questions (4 Prospective/2 Retrospective) 4 Debt Questions	<i>Random Order*</i>

* In each savings round, all prospective and retrospective questions were taken together. The order of question and question type were randomized. The 2 redundant savings questions were always taken last in round 1.

4.3 Incentives

Participants were given 8.00€ for showing up and completing the experiment. An additional payment ranging from 1.00-10.00€ was given to each participant based on the accuracy of a randomly chosen question. In order to avoid any unnecessary confusion by the participant, the instructions provided in the introduction of the experiment simply stated that (translated to English):

“An additional variable amount will be paid out based on the accuracy of one randomly generated question in the first two sections of the experiment. The question for this additional payout will be determined randomly after the experiment, so think carefully about all of your answers. To determine the question for your additional payout, you will draw a ball out of an urn with 31 balls (numbered 1 to 31). The additional variable payout ranges from **1,00€ to 10,00€**, with an average payout of **5,00€**.”

Upon completing the experiment, each subject randomly drew a numbered ping pong ball out of a covered box which denoted which question (out of the 31 total questions in the experiment)

was to be paid out.¹⁵ In total, participants were given 9.00-18.00€ for participating in the experiment, with an average payout of 11.76€.

5. Results – Experiment 1

5.1 Savings Domain

At the overall group level in the savings domain when calculators are prohibited (round 1), shown in Table 2, we find statistically significant exponential growth bias in all but one of the thirteen questions, using the Wilcoxon sign-ranked test.¹⁶ Using the Shapiro-Wilk test of normality, we find that the results are not normally distributed. Therefore, we display median figures and execute non-parametric statistical tests in our analysis. In this round, 77.8% of the questions were underestimated with a 0.86 median θ for all savings questions. The median answers of the initial nine prospective questions, displayed in Figure 1, suggests that the overall group not only demonstrates a “tendency to linearize”, but also takes an apparent additional step that adjusts the linear estimate upwards by roughly 10% across the various parameters of the experimental design. We are not sure if there is a systematic adjustment, but could find it plausible that individuals realizing a need to adjust their linear estimate upwards, used a simple heuristic, such as 10%, to derive their estimates. In some parameters with lower interest rates and/or time frames, this adjustment could be sufficient to eliminate the bias. On the other hand, this adjustment can substantially underestimate other compound growth scenarios, particularly those with longer time horizons and higher rates of returns.

Round 2 answers, with an available calculator, show that approximately 42% of the questions were answered correctly, posting answers within \$1 of the actual answer. However, the overall

¹⁵ This payout was formally derived by the answers θ output and calculated as: $10.00\text{€} - \text{abs}(\theta) * 10.00\text{€}$. If the payout is less than 1.00€, the minimum additional payment of 1.00€ was paid.

¹⁶ Using the Shapiro-Wilk normality test, we find that all bias results are not normally distributed.

group sample remains significantly biased for all questions in this round, although the median θ for all savings questions is 0.00.

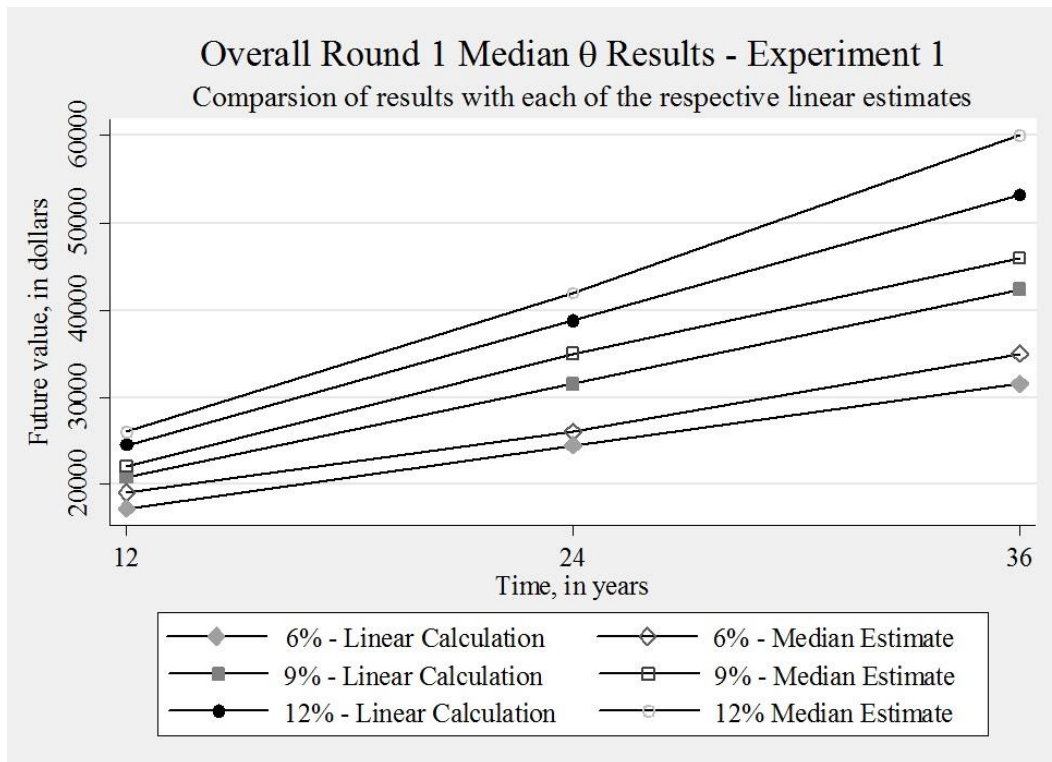
Table 2. Overall group level descriptive statistics for savings questions – Experiment 1, sorted by question. Note: the prospective question starts with an initial amount of \$10,000 and the retrospective question asks how much money one needs today in order to achieve the savings goal of \$100,000 in x years. Interest rate and years are listed below, respectively.

Results Summary		Round 1 (without calculator)				
Question	N	Med. θ	Biased	Linear	Median	Actual
Prospective; 0.06; 12	271	0.34***	67.5%	\$17,200	\$19,000	\$20,122
Prospective; 0.09; 12	272	0.80***	77.6%	\$20,800	\$22,000	\$28,127
Prospective; 0.12; 12	272	0.85***	76.5%	\$24,400	\$26,000	\$38,960
Prospective; 0.06; 24	272	0.85***	80.9%	\$24,400	\$26,000	\$40,489
Prospective; 0.09; 24	271	0.87***	87.1%	\$31,600	\$35,000	\$79,111
Prospective; 0.12; 24	271	0.93***	90.4%	\$28,800	\$42,000	\$151,786
Prospective; 0.06; 36	271	0.88***	86.0%	\$31,600	\$35,000	\$81,473
Prospective; 0.09; 36	271	0.94***	93.4%	\$42,400	\$46,000	\$222,512
Prospective; 0.12; 36	273	0.94***	94.9%	\$53,200	\$60,000	\$591,356
All Prospective	2444	0.89***	83.8%			
Retrospective; 0.06; 12	272	0.04***	54.4%	\$58,140	\$50,000	\$49,697
Retrospective; 0.12; 12	272	0.49	59.2%	\$40,984	\$32,750	\$25,668
Retrospective; 0.06; 36	273	0.73***	64.8%	\$31,646	\$25,000	\$12,274
Retrospective; 0.12; 36	271	0.90***	79.7%	\$18,797	\$15,000	\$1,691
All Retrospective	1088	0.62***	64.4%			
All Savings	3532	0.86***	77.8%			
Results Summary		Round 2 (with calculator)				
Question	N	Med. θ	Correct ^o	St. Dev		
Prospective; 0.06; 12	272	0.00***	48.5%	2.58		
Prospective; 0.12; 12	273	0.00***	46.2%	1.30		
Prospective; 0.06; 36	273	0.00***	42.9%	0.89		
Prospective; 0.12; 36	273	0.07***	42.1%	0.65		
All Prospective	1091	0.00***	44.9%	1.55		
Retrospective; 0.06; 12	272	0.00***	34.6%	5.74		
Retrospective; 0.12; 36	272	0.00***	36.7%	0.73		
All Retrospective	544	0.00	35.7%	4.24		
All Savings	1635	0.00***	41.8%	2.80		

^o Answers within \$1 +/- of the answer using the compound interest formula

Statistical significance based on the Wilcoxon signed-rank test - * 90%; ** 95%; *** 99%

Figure 1. Comparison of the overall round 1 median estimates with the linear calculation for each question - Experiment 1, prospective questions. Nine prospective savings questions were given in the absence of a calculator in a three by three (yearly interest rate by time in years) vector.



For the next step of the analysis, we classify those participants who are able to consistently calculate the savings questions in round 2 as the “capable with a calculator” group. Participants are placed in this group if they correctly answer three or more of the six savings questions in round 2 with a calculator. An answer is deemed “correct” if the posted answer is within \$1 of the actual answer. In this sample, 126 of the 273 (46.2%) participants qualify in this “capable with a calculator” group. In round 1, the prohibited calculator treatment, we find evidence that this “capable” group still possesses significant EGB as we reject the null hypothesis for **H1** ($p < .01$). Table 3 shows that 81.8% of the individuals in this group post an individual θ greater than 0.00. These results demonstrate that the correct calculation with a calculator does not totally de-bias individuals in the direct application in the same domain when calculators are prohibited.

Nevertheless, we would like to determine if the “capable with a calculator” group can at least provide significantly less biased estimates than those who are “incapable with a calculator”. In Table 3, we compare the difference of these two groups by testing **H2**, and can reject the null hypothesis at the 99% significance level. Here, we find evidence that the “capable with a calculator” group is significantly less biased in the savings domain without a calculator, posting a median θ of 0.78 compared to the “incapable with a calculator” group, who post a higher median θ of 0.98. These results suggest an important additional benefit of learning the proper application of the formal compound formula: the improvement in bias size when calculators are prohibited demonstrates that this learning may generate a more general and intuitive understanding of exponential effects. In the second round, the “incapable with a calculator” group made a slight median θ improvement (0.08) in estimates, but did not significantly improve their exponential growth bias with an available calculator. These results indicate that an available calculator does not assist individuals who are unable to apply to the correct calculation in making less biased estimates.

Table 3. Individual results summary in the savings domain – Experiment 1, by round and participant. Individual results are recorded as the median θ for all answers for each participant. A participant with a median $\theta > 0$ is shown to possess some level of EGB. Participants were placed in the capable with calculator group (“Capable with a Calc.”) by correctly calculating three or more of the six savings questions, within \$1 of the actual answer, with an available calculator.

Group	Round 1			Round 2			Difference	
	N	Med. θ	$\theta > 0$ (%)	N	Med. θ	$\theta = 0$ (%)	Med. θ	Z-Score
Incapable with a Calc.	147	0.98***	87.8%	147	0.90***	4.8%	0.08	1.52
Capable with a Calc.	126	0.78***	81.8%	126	0.00	99.2%	0.78***	10.02
Difference (z-score) H2		0.20***	(3.36)		0.90***	(10.09)		

Statistical significance of Wilcoxon signed-rank test - * 90%; ** 95%; *** 99%

Statistical significance of differences using the Wilcoxon rank-sum test - * 90%; ** 95%; *** 99%

5.2 Debt Results

In the slightly more complicated debt domain where the actual amortization formula was unknown by all participants, we can reject the null hypothesis for **H4** at the 99% statistical significance level and find a strong amortization bias in both rounds at the overall level. As depicted in Table 4, over 90% of all answers underestimate the remaining balance on a loan. The median estimate for all debt questions is slightly above the linear estimate, registering a median θ of 0.95. In this domain, we can observe that calculators do not provide any assistance in making better estimates, where slightly more estimates are positively biased (93.1%) compared to the prohibited calculator treatment (90.7%).¹⁷ At the overall level, median θ increases to 1.00 in round 2, from 0.95 in round one, although the differences are not statistically significant.

Table 4. Overall debt results summary - Experiment 1, sorted by question and round. All questions were for 30-year loans, with an original loan amount of \$100,000. 0.06 and 0.10 indicate the annual interest rate on the loan. 25%, 50%, and 75% indicate the remaining time on the loan (as a percentage).

Question – Rd.	N	Med. θ	St. Dev	Linear	Median	Correct	Under-est. (%)
30 Yr.; 0.06; 75%	268	1.00***	0.94	\$75,000	\$75,000	\$88,720	90.4%
30 Yr.; 0.06; 50%	269	0.91***	1.03	\$50,000	\$52,500	\$71,049	91.1%
30 Yr.; 0.06; 25%	273	0.93***	1.44	\$25,000	\$26,500	\$43,366	81.8%
30 Yr.; 0.10; 75%	273	1.00***	0.58	\$75,000	\$75,000	\$94,105	94.9%
30 Yr.; 0.10; 50%	271	0.93***	0.42	\$50,000	\$53,500	\$81,665	97.8%
30 Yr.; 0.10; 25%	272	0.94***	1.19	\$25,000	\$27,200	\$55,409	87.6%
All - Round 1	1626	0.95***	0.99				90.7%
30 Yr.; 0.06; 75%	271	1.00***	0.90	\$75,000	\$75,000	\$88,720	93.0%
30 Yr.; 0.06; 50%	270	1.00***	0.86	\$50,000	\$50,132	\$71,049	91.5%
30 Yr.; 0.10; 75%	270	1.00***	0.65	\$75,000	\$75,000	\$94,105	92.6%
30 Yr.; 0.10; 50%	273	0.92***	0.55	\$50,000	\$54,000	\$81,665	95.2%
All - Round 2	1084	1.00***	0.76				93.1%

Statistical significance of Wilcoxon signed-rank test - * 90%; ** 95%; *** 99%

¹⁷ If we compare only the round 1 estimates for the four questions given in round 2, 93.5% of the questions are underestimated.

When we evaluate the individual level results in the debt domain, shown in Table 5, no significant differences exist between rounds (calculator treatments). Again, at the individual level, we can reject the null hypothesis for **H4** and find a strong amortization bias in both rounds. However, in round 1, we detect a statistically significant difference of estimates between the “capable with a calculator” and “incapable with a calculator” groups. Here, we can reject the null hypothesis for **H5a**, as the “capable with a calculator” group provided less biased estimates than the “incapable with a calculator” group at the 99% significance level. These results suggest that the individuals in the “capable with a calculator” group appear to demonstrate a better grasp of general exponential effects not just in the savings domain, but also in a different and more complicated debt decision, when the actual formula was uniformly unknown.

Table 5. Individual results summary in the debt domain - Experiment 1, by round and participant. Individual results were recorded as the median θ for all relevant answers for each participant. A participant with a median $\theta > 0$ underestimated while those with a median $\theta < 0$ overestimated. Participants were placed in the "capable with a calc." group by correctly calculating three or more of the six savings questions, within \$1 of the actual answer, with an available calculator.

Group	N	Median θ	Z-Score	St. Dev	$\theta > 0$ (%)	$\theta < 0$ (%)
All Parts. - Round 1	273	0.94***	13.55	0.51	96.3%	3.7%
All Parts. - Round 2	273	1.00***	14.20	0.50	98.2%	1.8%
Difference		-0.06	-1.20			

Group	Round 1			Round 2			Difference	
	N	Med. θ	Z-Score	N	Med. θ	Z-Score	Med. θ	Z-Score
Incapable with a Calc.	147	1.00***	9.80	147	1.00***	10.43	0.00	-0.36
Capable with a Calc.	126	0.88***	9.40	126	0.97***	9.65	-0.09	-1.42
Difference	H5a.	0.12***	3.02	H5b.	0.02	1.58		

Statistical significance of Wilcoxon signed-rank test - * 90%; ** 95%; *** 99%

Statistical significance of differences using the Wilcoxon rank-sum test - * 90%; ** 95%; *** 99%

In round 2 with an available calculator in the debt domain, we cannot reject the null hypothesis for **H5b**, as the differences of estimates between these two groups were not significantly different. In this round, we observe an inordinately large number of participants posting answers outside of the sensible range of \$0 to \$100,000. Most of the “insensible” answers given were well above the initial amount of the loan. Upon a closer investigation of these answers, we find that 24 participants applied a variation of the compound savings equation, which is calculated as $B = A(1 + i)^N * \left[\frac{N-n}{N} \right]$, resulting in estimates that were well above the initial loan balance. For example, when we asked for the remaining balance of a 30-year, \$100,000 initial loan with a 6% annual interest rate after making payments for 15 years, these individuals posted: $\$287,174.56 = \$100,000(1 + .06)^{30} * \left[\frac{360-180}{360} \right]$. As shown in Table 6, we classify those 24 participants who mistakenly used this variation of the compound formula at least one time in the second round of the debt domain as “formula users”. When these participants eventually provided sensible answers, they gave significantly more biased estimates with calculators (median difference of 0.13) compared to their first round estimates. At the same time, this group of “formula users” provided significantly less biased estimates in the first round when a calculator was prohibited, by 0.10, compared to the group that didn’t attempt to apply a variation of this formula in the debt domain. For these participants, it appears that an available calculator doesn’t always decrease bias size. In fact, it may have hindered their ability to make a better estimate, as they incorrectly forced the compound interest formula into the wrong application.

Table 6. Comparing debt estimates of participants who mistakenly used a variation of the savings formula in the debt domain - Experiment 1. Participants are classified as "Formula Users" if they provided an insensible answer demonstrating use of the compound savings formula in the debt domain at least one time in the second round debt domain. The equation that we assume these participants applied is written formally as $B = [A(1+i)]^N \cdot [(N-n)/N]$.

Sub-Group	Round 1			Round 2			Difference	
	N	Med. θ	Z-Score	N	Med. θ	Z-Score	Med. θ	Z-Score
Non-Formula User	1483	0.97***	27.98	989	1.00***	25.21	-0.03	-1.51
Formula User	143	0.87***	6.80	95	1.00***	7.60	-0.13***	-2.63
Difference		0.10***	3.16		0.00	-0.44		

Statistical significance based on the Wilcoxon signed-rank test - * 90%; ** 95%; *** 99%

Statistical significance of differences using the Wilcoxon rank-sum test - * 90%; ** 95%; *** 99%

5.3 Conclusion and Discussion – Experiment 1

In this initial experiment, we find mostly encouraging evidence that speaks in favor of formal formula learning. Here, we find that the “capable with a calculator” individuals provided significantly less biased savings estimates without a calculator compared to the “incapable with a calculator” group. Additionally, this “capable with a calculator” group of participants posted less biased estimates in the debt domain when estimates were elicited in the absence of calculators. These findings suggest that the retention of the formal formula learning may not only assist participants in de-biasing individuals in the savings domain with an available calculator, it may also aid in developing a general or intuitive grasp of exponential effects not only in the same domain/direct area of application, but also in other exponentially-based domains. On the other hand, we find that too much dependence on these formulas can have adverse effects, as a number of participants who knew the compound savings formula mistakenly applied a variation of it in the debt domain leading to insensible answers well above the initial loan balance. Even when the experimental design forced these participants into giving answers at or below the initial loan balance, these individuals gave more biased answers than they provided without an available calculator.

These initial results mainly speak in favor of formal formula learning to effectively eliminate or reduce exponential growth bias in various savings and debt decision. In practice, however, most household savings and debt questions, such as those tested in Experiment 1, are generally applied quite infrequently. Thus, the effectiveness of the formula learning shown in the first experiment could remain limited if it cannot be adequately retained over time. Therefore, we seek to take an additional step in determining the “stickiness” of this formula learning by running a second experiment testing a specialized group of participants who have extensively learned about discounting and the compound savings formula approximately 18-20 months prior to completing our experiment.

6. Experiment 2

6.1 Motivation and Hypotheses

To the best of our knowledge, it has never been tested whether memorizing the general compound interest formula would be retained long enough to eliminate or reduce EGB over time. Most literature on improving financial literacy and personal finance education, as summarized by Fernandes, Lynch and Netemeyer (2014), shows that even extensive interventions do not assist in better decision making over time, nor does the availability of a calculator have any significant effects on savings estimates. McKenzie and Liersch (2011) previously allowed one treatment to use calculators, and found no difference in bias in the savings domain compared to the group without access to an available calculator and other electronic devices. In our second experiment, we employ a unique subject pool to test the long-term retention of the formula learning: 4th semester undergraduate business students at a top German university. These students had to pass a mandatory course in their 1st semester which extensively dealt with exponential growth and discounting. In this experiment, we test this group on similar savings and debt questions exclusively with an available calculator. In this sample, we anticipate:

H6 (Effective “stickiness” of the formula learning in the savings domain): The majority of participants will be able to retain their previous learning and calculate the correct answer. Thereby, these individuals will not show an exponential growth bias in the savings domain with an available calculator.

6.2 Experimental Set Up

The second experiment involves 251 undergraduate business students (128 males and 123 females) who were enrolled in a Corporate Finance class at the University of Muenster. The median participant age was 23 years, ranging from 19-27. The experiment was conducted in a computer lab and was fully set up in English, although a German translation of the main questions was also provided in the experiment.¹⁸ The overall study was not only aiming to answer the research question outlined in this paper, but it also intended to evaluate various learning methods to de-bias participants in the debt domain. Therefore, this experiment consisted of three stages. Upon completing various debt and savings questions in stage 1, participants were exposed to different task-specific tutorials and retested both immediately after taking the tutorial (stage 2) and three weeks later (stage 3). For the purposes of clearly examining the significant messages all three stages of the experiment, we will only consider the savings results from stage 1 in this paper. Findings from stages 2 and 3 regarding the effectiveness of various types of learning methods on reducing the amortization bias are discussed in Foltice (2015).

6.3 Incentives

For this experiment, each participant was given a base amount of 15.00€ for showing up to the first two stages, which lasted a total of 90-120 minutes. Additional variable payouts of 20.00€,

¹⁸ Since most of the courses in the Bachelors business program in Muenster are taught in English, we can safely assume that the German participants had little trouble understanding the English instructions.

40.00€, and 60.00€ were given to three randomly chosen participants out of the 20-25 individuals in each session. The additional payouts were determined by the overall average accuracy (absolute error %) of all questions in stages 1 and 2, compared to the other chosen participants in the group. Participants were informed upfront that their expected payment was monotonic in the accuracy of their estimates. Given the complexity of the mechanism, we refrained from providing more details about the payment structure.

6.4 Procedure

In Experiment 2, each participant received a total of sixteen questions, shown in Table 7, consisting of eight debt questions and eight savings questions (screenshots of the experiment are shown in Supplement A). The savings questions were further divided into four prospective savings questions and four retrospective savings questions.¹⁹ The order of the debt and savings questions was randomized, with half of the participants' receiving savings questions first and the other half receiving the debt questions first.²⁰

Table 7. Savings question vector - Experiment 2. Details of the savings questions given to each participant in Experiment 2.

Savings Questions (8)		
Prospective (4)	Annual Interest Rate	Time (in years)
Initial balance of \$10,000	7%	12
	12%	36
Retrospective (4)	Annual Interest Rate	Time (in years)
Savings goal of \$100,000	7%	12
	12%	36

In the savings domain, four of the 251 participants are immediately eliminated from the data set for completing the eight savings questions in a median time of five seconds or less.²¹ We

¹⁹ We again used a question structure in the savings domain that is consistent with Eisenstein and Hoch (2007). Here, a two-by-two matrix was chosen for the prospective and retrospective questions, consisting of slightly different parameters from Experiment 1: 7% and 12% yearly interest lasting 12 and 36 years.

²⁰ In the savings domain, the prospective and retrospective questions were grouped together.

²¹ This filter is consistent with Eisenstein and Hoch (2007) and was also checked in Experiment 1, though no participants were affected.

additionally eliminate all completely insensible answers, i.e. answers less (more) than the initial balance (savings goal) in the prospective (retrospective) questions. Unlike Experiment 1, this experimental design did not post an error message for these “insensible” answers. Consequently, seven additional participants are completely eliminated from the savings data set for posting more than three insensible answers.

6.5 Results - Experiment 2

Table 8 examines the results on individual question level using all “sensible” answers from the 240 remaining individuals in the dataset. Across all eight savings questions, 90.1% were answered correctly.²² The median θ is 0.00 for each of the eight savings questions.

The results shown in Table 8 also shed light on the exponential growth bias at the individual level, where the median bias “ θ ” is derived from all valid answers given by a specific individual. 95% of all participants (228 out of 240) produce a median bias of exactly 0.00. Only nine participants exhibit a median θ greater than zero, while the remaining three participants have a θ of less than zero. Only nine participants out of 240 provided zero correct answers, and 90% of all participants posted five or more (out of 8) correct answers. The median bias of all individual participants was again 0.00. Based on a binomial probability test for **H6**, we find strong evidence that the majority of participants shows no exponential growth bias as the null hypothesis can be rejected at the 99% confidence level.

²² A “correct” answer is defined as answers within \$1 greater or less than the answer generated by using the compound interest formula.

Table 8. Overall group level and individual level result summaries for savings questions - Experiment 2, sorted by question and individual. Note: the prospective question starts with an initial amount of \$10,000 and the retrospective question asks how much money one needs today in order to achieve the savings goal of \$100,000 in x years. Interest rate and years are listed below, respectively. Individual results are sorted by each participant's median θ for all eight savings questions.

Group Level Results Summary				Individual Level Results Summary	
Question	N	Median θ	Correct[°]		
Prospective; 0.07; 12	229	0.00	93.5%	N	240
Prospective; 0.07; 36	233	0.00	93.1%	Median θ	0.00
Prospective; 0.12; 12	238	0.00	92.2%	Mean	0.02
Prospective; 0.12; 36	238	0.00	88.7%	St. Dev.	0.23
All Prospective	938	0.00	91.8%	Minimum	-0.22
Retrospective; 0.07; 12	237	0.00	85.6%	Maximum	3.25
Retrospective; 0.07; 36	232	0.00	87.5%	Median $\theta = 0$; Unbiased	228
Retrospective; 0.12; 12	238	0.00	88.7%	Median $\theta > 0$; Biased	9
Retrospective; 0.12; 36	230	0.00	91.8%	2 or more (of 8) Correct	95.0%
All Retrospective	937	0.00	88.4%	5 or more (of 8) Correct	89.6%
All Savings	1875	0.00	90.1%	8 of 8 Correct	67.5%

[°] Denotes answers within \$1 +/- of the answer using the compound interest formula

Contrary to other attempts in improving financial decision making via increased financial education/literacy, we find evidence that formal formula learning can be widely retained over time. Presumably, these very precise estimates can be attributed to students knowing and computing the correct formula in the savings domain learned in their first semester Financial Mathematics course.²³ This is a testament to the intelligence of the students and provides encouraging evidence that extensively learning the actual equation can provide positive and durable effects for participants over time, at least in simple savings scenarios.

7. Discussion and Conclusion

Exponential growth effects play a major role in many household finance decisions and the consequences of a systematic bias in the decision-making process can lead to poor savings and debt decisions (Stango and Zinman, 2009). Our research contributes to the question of what

²³ We estimate that this learning occurred roughly 18-20 months before the experiment.

could be an effective way to help people eliminate or at least reduce the exponential growth bias (EGB) in various savings and debt decisions. In our main study (Experiment 1), we determine whether knowing the actual compound savings formula only assists in eliminating the exponential growth bias in its direct area of application with an available calculator, or if this knowledge extends into making less biased estimates in similar savings and debt decisions when either a calculator is prohibited or when the actual formula is not known. In the process of tackling this research question, we develop a measure for the exponential growth bias that naturally extends over different tasks and parameter settings. In Experiment 1, we find that the “capable with a calculator” individuals, who could later calculate the correct answers in the savings domain with a calculator, provide significantly less biased savings estimates without a calculator, compared to the “incapable with a calculator” group of participants that could not calculate the correct answers. Additionally, this group of “capable” participants provides less biased estimates in the debt domain when estimates were elicited in the absence of calculators. These findings suggest that the retention of the formal formula learning may not only assist in de-biasing individuals in the savings domain with an available calculator, but it may also aid in developing a more general and intuitive grasp of exponential effects not only in the same domain, but in other exponentially-based domains when the formula is not known. On the other hand, we find that too much dependence on these formulas can have adverse effects, as a number of participants who knew the compound savings formula mistakenly applied a variation of it in the debt domain leading to insensible answers well above the initial loan balance.

Based on the infrequency of applying such household savings and debt decisions, we also test if the formula learning can be effectively retained over time. In our follow-up study (Experiment 2), we test fourth semester business students who have previously learned about compound interest and discounting in a mandatory first semester class about 18 to 20 months beforehand. Here, we find evidence of the “stickiness” of the formula learning as a vast majority

of students (95%) are successful in consistently providing exact answers for the simple exponential savings scenarios with an available calculator.

Overall, we find strong evidence that speaks in favor of formal formula learning, particularly in the simple savings domain. In the U.S.A., a potential opportunity exists in introducing a more extensive learning curriculum to the current Common Core State Standards Initiative, which already expects everyone with an eighth grade education to be able to, or least be familiar with how to, calculate simple compound savings equations (Section 7.RP.A.3- L.12).²⁴ This shift could not only provide a cost effective way to reduce bias and improve financial savings and debt decisions over time, it could also be more easily implemented by middle school, high school and college business and math teachers who, presumably, should already feel comfortable teaching exponential compounding. This suggestion also eliminates a major “training the trainer” challenge of implementing financial literacy and personal finance training programs on a large scale, as most teachers do not feel adequately prepared to teach such courses (Way and Holden, 2009).

The main goal of this paper is to take initial steps into exploring appropriate learning methods that lead to an effective and retainable elimination or reduction of EGB in various savings and debt decisions. While the main findings in this paper lead us to suggest formal formula learning, we do not believe that this formal formula learning is a “one size fits all” solution for the elimination of EGB for all savings and debt decisions. In fact, Foltice (2015) confirms the complexity of this solution by providing evidence that experiential learning reduces the amortization bias over time in the debt domain more so than formula learning, even for numerically-minded students. We also can’t rule out other potential influences on the exponential growth bias size, such as financial literacy, numeracy proficiency, or the effects of previous experience with various savings and debt products. Foltice and Langer (2015) take a

²⁴ Viewed on 3/9/2015 at <http://www.ixl.com/standards/common-core/math/grade-7>.

deeper look into these aspects and address the various drivers of the EGB as it pertains to undergraduate students in the U.S.A. Finally, we don't know if this formal learning is the most effective learning method for all individuals, who possess different learning preferences and capacities. Whether EGB can be eliminated or reduced more effectively by providing visual aids, (simplified) formulas for the more complicated tasks, various heuristics such as the Rule of 70 or 72, or by feedback-based learning is an important follow-up question.

Appendix 1. Derivation of the equation (1) from the main text (for payment per period P and remaining balance B).

For an overall loan amount A, a payment P per period, and an interest rate i, the remaining balance B_n after n periods can be derived recursively as:

$$B_n = B_{n-1} \cdot (1 + i) - P.$$

Starting from

$$B_0 = A$$

we then obtain

$$B_n = A \cdot (1 + i)^n - P \cdot \sum_{t=0}^{n-1} (1 + i)^t$$

and

$$B_n = A \cdot (1 + i)^n - P \cdot \frac{(1+i)^n - 1}{(1+i) - 1}.$$

This gives equation (a):

$$B_n = A \cdot (1 + i)^n - \frac{P}{i} \cdot [(1 + i)^n - 1].$$

Equation (a) follows from the assumption that the loan is fully paid back after N periods, i.e.:

$$B_N = 0$$

This leads to:

$$A \cdot (1 + i)^N - \frac{P}{i} \cdot [(1 + i)^N - 1] = B_N = 0$$

and solving for P gives equation (b):

$$P = A \cdot i \cdot \frac{(1+i)^N}{(1+i)^N - 1} = A \cdot i \cdot \frac{1}{1 - (1+i)^{-N}}.$$

To derive equation (1) we substitute (a) into (b) and obtain:

$$B_n = A \cdot (1 + i)^n - \frac{P}{i} \cdot [(1 + i)^n - 1] = A \cdot (1 + i)^n - A \cdot \frac{[(1 + i)^n - 1] \cdot (1 + i)^N}{(1 + i)^N - 1}$$

Some algebra gives:

$$\begin{aligned} B_n &= A \cdot \frac{[(1 + i)^N - 1] \cdot (1 + i)^n}{(1 + i)^N - 1} - A \cdot \frac{[(1 + i)^n - 1] \cdot (1 + i)^N}{(1 + i)^N - 1} \\ &= A \cdot \frac{(1 + i)^N \cdot (1 + i)^n - (1 + i)^n}{(1 + i)^N - 1} - A \cdot \frac{(1 + i)^n \cdot (1 + i)^N - (1 + i)^N}{(1 + i)^N - 1} \\ &= A \cdot \frac{-(1 + i)^n}{(1 + i)^N - 1} - A \cdot \frac{-(1 + i)^N}{(1 + i)^N - 1} = A \cdot \frac{(1 + i)^N - (1 + i)^n}{(1 + i)^N - 1} \\ &= A \cdot \left[1 - \frac{(1 + i)^n - 1}{(1 + i)^N - 1} \right] \end{aligned}$$

Appendix 2. Extended thoughts about an appropriate accumulation factor $f_{i,t}(\theta)$

We have to consider two functions:

1. The function $f_{i,t}(\theta)$ that is the accumulation factor itself, at the same time it is the factor that solely determines the effects in the savings scenarios by the relation $FV = PV \cdot f_{i,t}(\theta)$. We measure bias size by $\theta = f_{i,t}^{-1}\left(\frac{FV}{PV}\right)$ in the savings case.
2. The function $g_{i,n,N}(\theta)$ that determines the effects in the amortization scenario via the formula for the remaining balance: $B = A \cdot g_{i,n,N}(\theta)$. It holds: $g_{i,n,N}(\theta) = \frac{f_{i,N}(\theta) - f_{i,n}(\theta)}{f_{i,N}(\theta) - 1}$ if we replace all exponential terms by $f_{i,t}(\theta)$. We measure bias size by $\theta = g_{i,t}^{-1}\left(\frac{B}{A}\right)$ in this case.

The function $f_{i,t}(\theta)$ needs to have some “nice properties”, not only to make it itself suitable for the savings scenario but also to make the derived $g_{i,n,N}(\theta)$ suitable for the amortization scenario. Both functions should be monotonic (in the relevant) range, they should be “well calibrated” and they should be able to assign θ s to all reasonable answers for the given task.

We consider four different functional forms for the function $f_{i,t}(\theta)$:

- A. The functional form $f_{i,t}(\theta) = (1 + i)^{(1-\theta)t}$, previously used in the literature, can be considered inappropriate, because it is only calibrated for “perfect exponential” behavior but not for perfect linear behavior (perfect linear behavior results in different θ for different i and t .)
- B. The functional form $\hat{f}_{i,t}(\theta) = \theta \cdot (1 + t \cdot i) + (1 - \theta) \cdot (1 + i)^t$ is more appropriate, because it is calibrated both on “perfect exponential” estimate ($\theta=0$) and for perfect linear estimate ($\theta=1$). It is also nice with respect to its “axiomatic foundation”. It follows from a development of the exponential term as a sum: $(1 + i)^t = \sum_{j=0}^t \binom{t}{j} i^j = (1 + t \cdot i) + \sum_{j=2}^t \binom{t}{j} i^j$ and an underweighting of the higher order components: $\hat{f}_{i,t}(\theta) = (1 + t \cdot i) + (1 - \theta) \cdot \sum_{j=2}^t \binom{t}{j} i^j$.

The function $\hat{f}_{i,t}(\theta)$ has one problem, however. The derived $g_{i,n,N}(\theta)$ has the unattractive property that the function is not able to assign θ s to all reasonable answers to the task. It can be shown that it holds: $\lim_{\theta \rightarrow \infty} g_{i,n,N}(\theta) = 1 - \frac{(1+i)^n - n \cdot i}{(1+i)^N - N \cdot i} < 1$. For example, if $i=10\%$, $N=20$, and $n=5$, we have $\lim_{\theta \rightarrow \infty} g_{i,n,N}(\theta) = 1 - \frac{0.1105}{3.7275} = 0.97035$. Therefore, for $A=\$200,000$, any $B > \$194,070$ cannot be assigned a θ . The problematic range becomes larger for smaller i and for decreasing $\frac{N-n}{N}$.

- C. It can be shown that the problem arises because of the linearity of the function $\hat{f}_{i,t}(\theta)$ in θ . We need a function $f_{i,t}(\theta)$ that is convex in θ , to receive a reasonable range for the values of $g_{i,n,N}(\theta)$.

This leads to the function: $\dot{f}_{i,t}(\theta) = (1 + t \cdot i)^\theta \cdot (1 + i)^{(1-\theta) \cdot t}$. This function is interesting for various reasons:

- i. It is just an extension of the previously used form (see A.)
- ii. It has all the nice properties for $f_{i,t}(\theta)$ itself: it is calibrated for both the “perfect exponential” ($\theta=0$) and the perfect linear estimate ($\theta=1$).
- iii. $f_{i,t}(\theta)$ assumes values in $(0, \infty)$, which is no relevant restriction at the lower boundary, because answers FV of 0 or smaller would be considered “confused” anyway.
- iv. It assumes reasonable values of $g_{i,n,N}(\theta)$ and maps to the complete range $[-\infty, 1]$. Here, we have a restriction that answers of $B \geq A$ could not be transformed into a θ , but this also seems to be a reasonable restriction (it can be argued again that $B > A$ hints to a confused participant anyway).

The only detriment of this measure is that has less of an intuitive component (“what part of the higher order interest is considered”). The function is also not continuous. It has a jump at $\frac{\ln(1+iN)}{\ln(1+iN) - N \cdot \ln(1+i)}$. To give an example: for the parameters $i=10\%$ and $N=20$ no θ -values above 2.36 can be assumed.

An interesting insight is generated by writing the function $\dot{f}_{i,t}(\theta) = (1 + t \cdot i)^{\theta} \cdot (1 + i)^{(1-\theta) \cdot t}$. $(1 + i)^{t \cdot \theta}$ as $\dot{f}_{i,t}(\theta) = (1 + t \cdot i) \cdot e^{\ln\left(\frac{(1+i)^t}{1+ti}\right) \cdot \theta}$, because it shows that $\dot{f}_{i,t}(\theta)$ is simply an exponential function that is the only member of the functional family $f(\theta) = a \cdot e^{b \cdot \theta}$ that has the required properties: $f(1) = (1 + i \cdot t)$ and $f(0) = (1 + i)^t$. This derivation leads to a further, even better suited functional type:

- D. If we set up the functional family as $f(\theta) = a \cdot e^{b \cdot \theta} + 1$ and fit it to the two required data points, we have a more reasonable limit case. It holds that: $\lim_{\theta \rightarrow -\infty} a \cdot e^{b \cdot \theta} + 1 = 1$. Thereby we have an even more reasonable limit for sensible answers in the savings scenario. It is insensible to give an answer for FV that is not larger than PV. Any $f(\theta) \leq 1$ can be considered to be a confused answer [in the same way as any answer of $B \geq A$ can be considered a confused answer in the amortization case. If we fit the functional form to the two data points, we obtain:

$$\check{f}_{i,t}(\theta) = (t \cdot i)^\theta \cdot ((1 + i)^t - 1)^{(1-\theta)} + 1$$

Appendix 3. Savings and debt question vector – Experiment 1. Details of the savings and debt questions given to each participant in both rounds of the experiment.

Savings Questions (15) - Round 1			
Prospective (9)	Annual Interest Rate		Time (in years)
Initial balance of \$10,000	6%		12
	9%		24
	12%		36
Retrospective (4)	Annual Interest Rate		Time (in years)
Savings goal of \$100,000	6%		12
	12%		36
Redundant Prospective (2)	Annual Interest Rate		Time (in years)
Initial balance of \$10,000	Q1	6%	12
	Q2	12%	36
Debt Questions (6) - Round 1			
Long Term (6)	Annual Interest Rate		Time Remaining on Loan (%)
Initial 30-year loan of \$100,000	6%		75%
			50%
	10%		25%
Savings Questions (6) - Round 2			
Prospective (4)	Annual Interest Rate		Time (in years)
Initial balance of \$10,000	6%		12
	12%		36
Retrospective (2)	Annual Interest Rate		Time (in years)
Savings goal of \$100,000	Q1	6%	12
	Q2	12%	36
Debt Questions (4) - Round 2			
Long Term (4)	Annual Interest Rate		Time Remaining on Loan (%)
Initial 30-year loan of \$100,000	6%		75%
	10%		50%

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Supplement A. Experiment Information (Experiment 2)

Experiment Introduction and Instructions

Experiment - Stages 1 & 2

Finance Center Münster
Lehrstuhl für Finanzierung

Login

Please enter your personal data and click on "Continue" to continue with the experiment.

First name:


Last name:

Matrikelnummer:

Your personal data is needed in order to identify you for your payout. The following experiment data is only used anonymously for research purposes.

Continue

Experiment - Stages 1 & 2

Finance Center Münster
Lehrstuhl für Finanzierung

Welcome to Stages 1 and 2 of this experiment.

The overall experiment consists of 3 Stages. Stages 1 and 2 will be completed today (your appointment 1) and Stage 3 (your appointment 2) will be completed at a later date. Today's experiment will take approximately 2 hours to complete, about an hour for Stage 1 and an hour for Stage 2.

In about three weeks, Stage 3 (your appointment 2) will take approximately 45 minutes to complete.

Thank you for participating in this experiment. Your assistance is greatly appreciated.

Continue

Experiment Introduction and Instructions (continued)

Experiment - Stages 1 & 2

Finance Center Münster
Lehrstuhl für Finanzierung

Incentives

Today, each participant will be given a base amount of €15 for completing Stages 1 and 2.

An additional variable amount will be paid out to 3 randomly chosen participants from each session (i.e. the people sitting in this room) with payouts consisting of €20.00, €40.00 and €60.00. The variable payout will depend on the overall average accuracy of your answers/estimates in both stages, if chosen, compared to other chosen participants in this session.

The overall payout range today will be €15 to €75, with an average approximate €20 payout for each participant.


For your second Appointment (Stage 3), you will be given an additional base fee as well as possible additional variable payouts for completing the final Stage.

For the purposes of this experiment, it is essential that you show up for your second appointment.

Back

Continue

Experiment - Stages 1 & 2

Finance Center Münster
Lehrstuhl für Finanzierung

Please follow the provided instructions and give your best estimate when necessary.

You may answer each question in whole numbers, for example \$51000, or in 2 decimal points, for example \$51000.34. You are not required to use a decimal point in your answers, but if you do, please use a '.' instead of a ','.

You may use a calculator, pencil/pen, and paper, which will be provided by the experimenter.

Please note: There is no 'back' button for this experiment. When you click 'continue', you can't go back.

Continue

Introduction/Incentives and Instructions (text)

Introduction – Stages One and Two

Welcome to Stages 1 and 2 of this experiment.

This experiment consists of 3 Stages. Stages 1 and 2 will be completed today and Stage 3 will be completed at a later date. It will take approximately 2 hours to complete Stages 1 and 2, about an hour for each Stage. Stage 3 will take approximately 45 minutes to complete.

Please follow the provided instructions and give your best estimate/guess when necessary.

You may use a calculator, pencil/pen, and paper, which will be provided by the experimenter.

Thank you for participating in this experiment. Your assistance is greatly appreciated.

Incentives - Stages one and two

Today, each participant will be given a base amount of €15 for completing Stages 1 and 2.

An additional variable amount will be paid out to 3 randomly chosen participants from each session (i.e. the people sitting in this room) with payouts consisting of €20.00, €40.00 and €60.00. The variable payout will depend on the overall average accuracy of your answers/estimates in both stages, if chosen, compared to other chosen participants in this session.

The overall payout range today will be €15 to €75, with an average approximate €20 payout for each participant.

For your second Appointment (Stage 3), you will be given an additional base fee as well as possible additional variable payouts for completing the final Stage.

For the purposes of this experiment, it is essential that you show up for your second appointment.

Final Instructions – All Stages (1-3)

Please follow the provided instructions and give your best estimate when necessary.

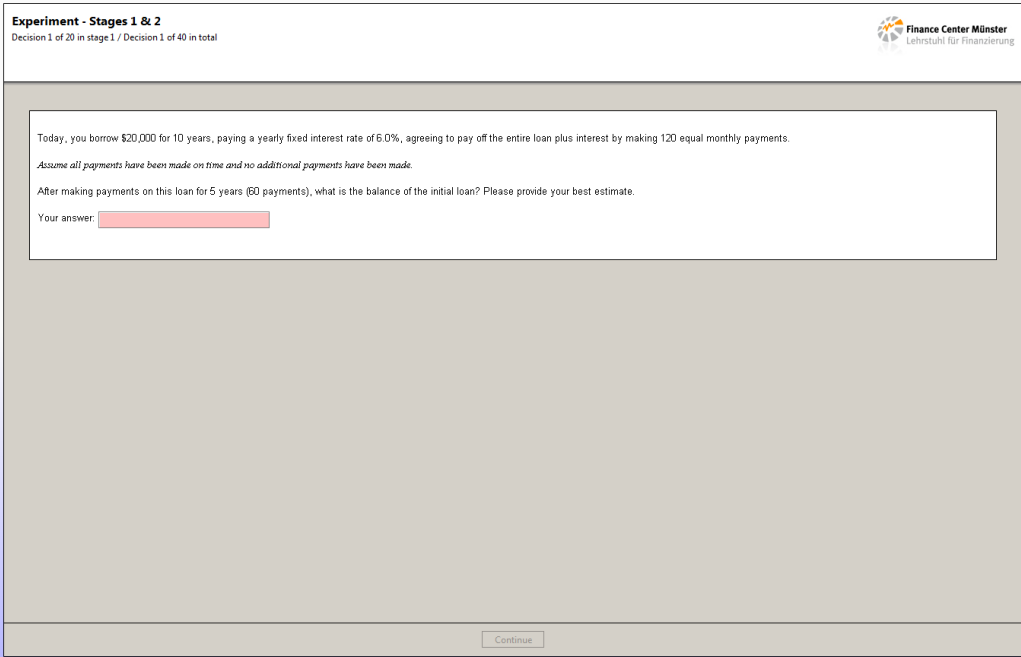
You may answer each question in whole numbers, for example \$51000, or in 2 decimal points, for example \$51000.34. You are not required to use a decimal point in your answers, but if you do, please use a ‘.’ Instead of a ‘,’

You may use a calculator, pencil/pen, and paper, which will be provided by the experimenter.

Please note: There is no ‘back’ button for this experiment. When you click ‘continue’, you can’t go back.

Click ‘continue’ to begin the next/final stage.

Experiment Debt Question Format



Experiment - Stages 1 & 2
Decision 1 of 20 in stage 1 / Decision 1 of 40 in total

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Lehrstuhl für Finanzierung

Today, you borrow \$20,000 for 10 years, paying a yearly fixed interest rate of 6.0%, agreeing to pay off the entire loan plus interest by making 120 equal monthly payments.
Assume all payments have been made on time and no additional payments have been made.
After making payments on this loan for 5 years (60 payments), what is the balance of the initial loan? Please provide your best estimate.

Your answer:

Continue

Text


Today, you borrow \$_____ for ____ years, paying a yearly fixed interest rate of _____%, agreeing to pay off the entire loan plus interest by making _____ equal monthly payments.

Assume all payments have been made on time and no additional payments have been made.

After making payments on this loan for _____ years (____ payments), what is the remaining balance of the initial loan? Please provide your best estimate.

Savings Question Format (Prospective)

Experiment - Stages 1 & 2
Decision 11 of 20 in stage 1 / Decision 11 of 40 in total

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You currently have a balance of \$10,000 in your account. You leave this money in your savings account for 36 years at a constant annual interest rate of 12%.

Assume no additional deposits or withdrawals. Interest is compounded annually and reinvested into the account.

Based on the above information, estimate your total account balance after 36 years. Please provide your best estimate.

Your answer:

Continue


You currently have a balance of \$10,000 in your account. You leave this money in your savings account for ___ years at a constant annual interest rate of _%.

Assume no additional deposits or withdrawals. Interest is compounded annually and reinvested into the account.

Based on the above information, estimate your total account balance after ___ years. Please provide your best estimate.

Savings Question Format (Retrospective)

Experiment - Stages 1 & 2
Decision 15 of 20 in stage 1 / Decision 15 of 40 in total

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Lehrstuhl für Finanzierung

Your goal is to have \$100,000 in your savings account 36 years from today. Today, you will invest an initial amount of money in your savings account for 36 years at a constant interest rate of 12% per year.

Assume no additional deposits or withdrawals. Interest is compounded annually and reinvested into the account.

How much do you need to invest today in order to reach your savings goal in 36 years?

Your answer:

Continue


Your goal is to have \$100,000 in your savings account ___ years from today. Today, you will invest an initial amount of money in your savings account for ___ years at a constant interest rate of _% per year.

Assume no additional deposits or withdrawals. Interest is compounded annually and reinvested into the account.

***How much do you need to invest today in order to reach your savings goal in ___ years?
Please provide your best estimate.***

Information and Conclusion

Experiment - Stages 1 & 2

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Age

Email address

Sex

☐ male

☐ female

What is your Major?

Are you an international student?


☐ Yes

☐ No

Which country are you from?

Continue

Experiment - Stages 1 & 2

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Thank you for participating in the experiment!

You can get your 15 € now from the experiment supervisor.

In addition, you have the chance for an extra payout. Good luck!

Continue